Wave Optics

INTERFERENCE Section - 1

The term 'interference', in general, refers to any situation where two or more waves overlap each other in same region of space. But usually, interference refers to the superposition of two waves of same frequency moving in same direction.

Consider the superposition of two waves of amplitudes A_1, A_2 and wavelength λ; at a point P. One wave covers distance x_1 from its source S_1 to point P and the other wave covers x_2 from its source S_2 to point P. The particle displacements at P are :

$$y_1 = A_1 \sin\left(\frac{2\pi t}{T} - \frac{2\pi x_1}{\lambda} + \phi_{s_1}\right) \quad ; \quad y_2 = A_2 \sin\left(\frac{2\pi t}{T} - \frac{2\pi x_2}{\lambda} + \phi_{s_2}\right)$$

where ϕ_{s_1} and ϕ_{s_2} are phases at $t = 0$ and $x = 0$ and entirely depend on the sources S_1 and S_2 because sources are placed at origins ;

S_1 at $x_1 = 0$ and S_2 at $x_2 = 0$

Let $\phi_1 = \frac{2\pi t}{T} - \frac{2\pi x_1}{\lambda} + \phi_{s_1}$ and $\phi_2 = \frac{2\pi t}{T} - \frac{2\pi x_2}{\lambda} + \phi_{s_2}$

Let $\Delta\phi = \phi_1 - \phi_2 = \frac{2\pi}{\lambda}(x_2 - x_1) + \phi_{s_1} - \phi_{s_2}$

$\Rightarrow$ $y_1 = A_1 \sin \phi_1$ and $y_2 = A_2 \sin(\phi_1 - \Delta\phi)$

$\Rightarrow$ the resultant wave is $y = y_1 + y_2 = A_1 \sin \phi_1 + A_2 \sin(\phi_1 - \Delta\phi)$

$\Rightarrow$ $y = (A_2 \cos \Delta\phi + A_1) \sin \phi_1 - (A_2 \sin \Delta\phi) \cos \phi_1$

$\Rightarrow$ $y = \left(\sqrt{(A_2 \cos\Delta\phi + A_1)^2 + A_2^2 \sin^2 \Delta\phi}\right) \sin\left[\phi_1 - \tan^{-1}\frac{A_2 \sin\Delta\phi}{A_1 + A_2 \cos\Delta\phi}\right]$

$\Rightarrow$ The resultant amplitude is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\Delta\phi} \quad \text{where} \quad \left[\Delta\phi = \frac{2\pi}{\lambda}(x_2 - x_1) + \phi_{s_1} - \phi_{s_2}\right]$$

$\Rightarrow$ The resultant amplitude depends on A_1, A_2 and $\Delta\phi$: the phase difference between waves.

- $x_2 - x_1 = p$ is known as path difference
- If $\phi_{s_1} - \phi_{s_2} = 2n\pi$, then $\Delta\phi \equiv \frac{2\pi p}{\lambda}$

  and sources are said to be in same phase.
- If $\phi_{s_1} - \phi_{s_2} = (2n+1)\pi$, then $\Delta\phi = \frac{2\pi p}{\lambda} + \pi$

  and sources are said to be in opposite phase

Constructive Interference :

If $\Delta\phi$ = even multiple of π, then $\cos\Delta\phi = 1$ and amplitude is maximum.

$\Rightarrow \quad A_{max} = A_1 + A_2$

Condition for constructive interference :

➤ For sources in same phase, $\Delta\phi = \frac{2\pi}{\lambda} p = 2n\pi$

$\Rightarrow \quad$ (for constructive interference.) p = integral multiple of λ

$\Rightarrow \quad p = n\lambda$

Destructive Interference :

If $\Delta\phi$ = odd multiple of π, then $\cos\Delta\phi = -1$ and amplitude is minimum.

$\Rightarrow \quad A_{min} = A_1 - A_2$

Condition for destructive interference :

➤ $\Delta\phi = \frac{2\pi}{\lambda} p = (2n-1)\pi$

$\Rightarrow \quad p = (\text{odd no.})\ \lambda/2 \qquad$ for destructive interference

$\Rightarrow \quad p = (2n-1)\ \lambda/2$

Important Note :

The above conditions are applicable for sources in same phase. If the sources are in opposite phase, the conditions for constructive and destructive interference are reversed.

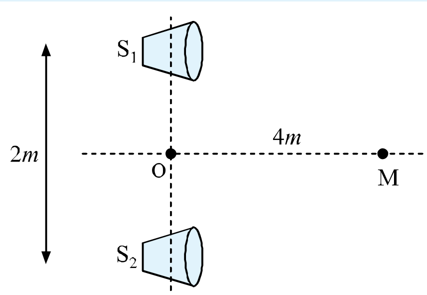
$\Rightarrow \quad$ For sources is opposite phase :

$p = n\lambda \qquad$ for destructive interference

$p = (2n-1)\ \lambda/2 \qquad$ for constructive interference.

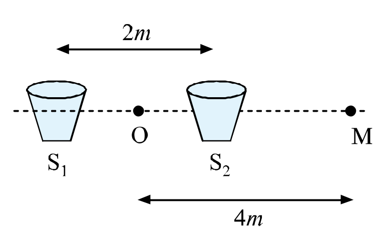
Illustration - 1 *Two spheres connected to the same source of fixed frequency are placed 2.0 m apart in a box. A sensitive microphone placed at a distance of 4.0 m from their midpoint along the perpendicular bisector shows maximum response. The box is slowly rotated till the speakers are in the same line with the microphone. The distance between the mid–point of the speakers and the microphone remains unchanged. Exactly 5 more maximum responses are heard in the microphone in doing this with the 5th maximum heard in the final orientation. Calculate the wavelength of the sound.*

SOLUTION :



Initial position

Path difference $= S_1M - S_2M = 0$



Final position

Path difference $= S_1M - S_2M = 2m$

Let $\lambda =$ wavelength.

$\Rightarrow$ For maximum response (constructive interference),

$$p = 0, \lambda, 2\lambda, 3\lambda, 4\lambda......$$

As the box is rotated slowly, the path difference increases from zero to two metres. Hence,

First response at $p = \lambda$, second at $p = 2\lambda$,......,

Fifth at $p = 5\lambda$.

Hence finally, $p = 5\lambda$

$\Rightarrow \quad 2 = 5\lambda \quad \Rightarrow \quad \lambda = 0.4.$

Illustration - 2 *Two point sources of radiation : S_1 and S_2 radiate waves of same frequency and are excited by the same oscillator. They are also in phase with each other. S_1 is placed at origin O while S_2 is placed at the point (0, 4) on Y-axis. Find the points of maxima of received intensity if a detector is moved towards positive X–axis starting from origin. The wavelength of radiation is 1m.*

SOLUTION :

Let the detector be at the point D (x, o).

Let $S_1S_2 = d = 4m$.

$\Rightarrow$ Path difference $= S_2D - S_1D$

$$P = \sqrt{x^2 + d^2} - x$$

For constructive interference, $P = n\lambda$

$$\Rightarrow \quad \sqrt{x^2 + d^2} - x = n\lambda$$

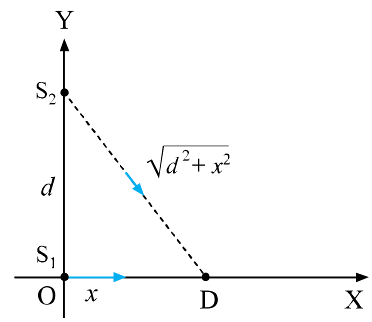
Using $d = 4m$, $\lambda = 1m$; $\sqrt{x^2 + 16} - x = n$

$$\Rightarrow \quad x = \frac{8}{n} - \frac{n}{2}$$

It is easily seen that the path difference decreases from $p \approx 4m$ to ($p \to 0$ as $x \to \infty$)

$\Rightarrow$ n can have values $n = 3, 2, 1$

Going away from O towards the X–axis,



- First maxima occurs at

$$x = \frac{8}{3} - \frac{3}{2} = \frac{7}{6}m \qquad (\text{for } n = 3)$$

- Second maxima occurs at

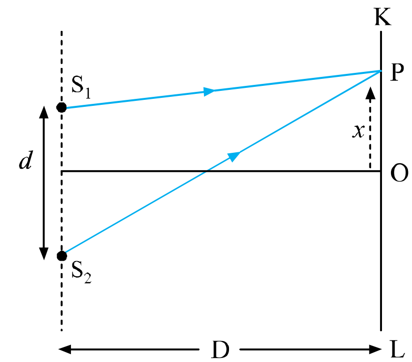
$$x = \frac{8}{2} - \frac{2}{2} = 3m \qquad (\text{for } n = 2)$$

- Third maxima occurs at

$$x = \frac{8}{1} - \frac{1}{2} = 7.5m \qquad (\text{for } n = 1)$$

YOUNG'S DOUBLE SLIT EXPERIMENT — Section - 2

Consider two coherent sources of light : S_1 and S_2 separated by a small distance d. KL is a screen placed at a distance D from the line joining S_1 and S_2. The light waves going from S_1 and S_2 to any point P on the screen interfere with each other. The points where the path difference is integral multiple of wavelength will be brightly illuminated because constructive interference occurs. Some points will be dark where destructive interference occurs. Thus the overall picture is a pattern of dark and bright bands known as fringe pattern. The dark bands are known as dark fringes and bright bands are known as bright fringes. The separation between two consecutive dark (or bright) fringes is known as fringe width (ω).



Taking O (on the screen and on the right bisector of $S_1 S_2$) as origin, we first find an expression for the path difference at an arbitrary point P located at coordinate x from the origin O.

It can be shown that path difference at P = $S_2P - S_1P = \frac{xd}{D}$

(Assuming $d << D$).

Location of Bright Fringes :

For constructive interference,

$$P = \frac{xd}{D} = 0, \lambda, 2\lambda, 3\lambda, \ldots\ldots$$

⇒ Bright fringes are located at

$$x = 0, \frac{D\lambda}{d}, \frac{2D\lambda}{d}, \frac{3D\lambda}{d}, \ldots\ldots .$$

⇒ Position of nth bright fringe is

$$x_n = \frac{nD\lambda}{d}$$

Location of Dark Fringes :

For destructive interference,

$$P = \frac{xd}{D} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots\ldots$$

Dark fringes are located at

$$x = \frac{\lambda D}{2d}, \frac{3\lambda D}{2d}, \frac{5\lambda D}{2d}, \ldots\ldots\ldots\ldots\ldots$$

⇒ Position of *n*th dark fringe is

$$x_n = (2n-1)\frac{D\lambda}{2d}$$

Fringe Width :

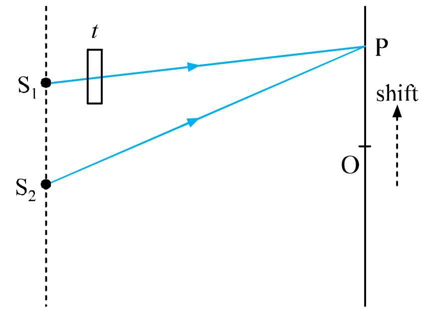
ω = distance between two successive bright (or dark) fringes

$\Rightarrow \quad \omega = \dfrac{D\lambda}{d}$

angular fringe width or angular separation between fringes is

$$\theta = \frac{\omega}{D} \Rightarrow \theta = \frac{\lambda}{d}$$

Displacement of Fringe Pattern :



Let us analyse what happens if a thin transparent plate of thickness t and refractive index m is placed in front of one of the sources, for example in front of S_1. This changes the path difference because light from S_1 now takes longer time to reach the screen.

$$\text{Time taken} = \frac{d_{air}}{v_{air}} + \frac{d_{plate}}{v_{plate}}$$

$$= \frac{S_1P - t}{c} + \frac{t}{c/\mu} \quad \text{[using the definition of refractive index } \mu_1 = \frac{c_{air}}{c_1}\text{]}$$

$$= \frac{S_1P + t(\mu - 1)}{c}$$

Hence the effective path that is equivalently covered in air is $S_1P + t(\mu - 1)$.

$\Rightarrow \quad$ Path difference $= S_2P - [S_1P + t(\mu - 1)]$

$$= S_2P - S_1P - t(\mu - 1)$$

$$= \frac{xd}{D} - t(\mu - 1)$$

$\Rightarrow \quad$ Points of bright fringes are located at x for which,

$$\frac{xd}{D} - t(\mu - 1) = 0, \lambda, 2\lambda, 3\lambda, \ldots\ldots\ldots$$

$$\Rightarrow \quad x = \frac{Dt(\mu - 1)}{d}, \frac{D\lambda}{d} + \frac{Dt(\mu - 1)}{d}, \ldots\ldots\ldots$$

Hence the bright fringes (and also the dark fringes) shift upward, (i.e. in direction of S_1) by a distance of $\dfrac{Dt(\mu - 1)}{d}$

$\Rightarrow \quad \text{shift} = \dfrac{Dt(\mu - 1)}{d}$

Intensity

Let I_1 and I_2 be the intensities of two waves interfering with each other.

Using the relation $I \propto A^2$, $I_1 \propto A_1^2$, $I_2 \propto A_2^2$;

The expression for resultant intensity is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi \quad \text{(where } \Delta\phi = \text{phase difference)}$$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \quad \text{for} \quad \Delta\phi = 0 \text{ or } 2n\pi$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \quad \text{for} \quad \Delta\phi = \pi \text{ or } (2n-1)\pi$$

Some more results :

(i) If the two waves interfering with each other have identical amplitudes and intensities, then we have

$$A_{max} = 2A_0,\ A_{min} = 0$$

where A_0 = amplitude of each wave

$$I_{max} = 4I_0,\ I_{min} = 0$$

I_0 = intensity of each wave

⇒ The expression for resultant amplitude and intensity are :

$$A = A_{max}\cos\frac{\Delta\phi}{2}$$

$$I = I_{max}\cos^2\frac{\Delta\phi}{2}$$

⇒ For sources in same phase,

$$A = A_{max}\cos\frac{\pi x}{\omega}$$

$$I(x) = I_{max}\cos^2\frac{\pi x}{\omega} \quad \text{(where } \omega = \text{fringe width)}$$

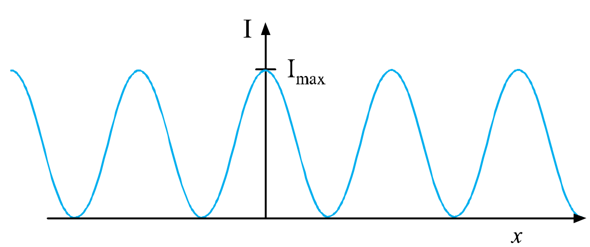
(ii) The ratio of maximum and minimum amplitudes is

$$\frac{A_{max}}{A_{min}} = \frac{A_1 + A_2}{A_1 - A_2} = \frac{r+1}{r-1} \quad \left(\text{where } r \text{ is the ratio of amplitudes } r = \frac{A_1}{A_2}\right)$$

The ratio of maximum and minimum intensities is $\frac{I_{max}}{I_{min}} = \left(\frac{r+1}{r-1}\right)^2$

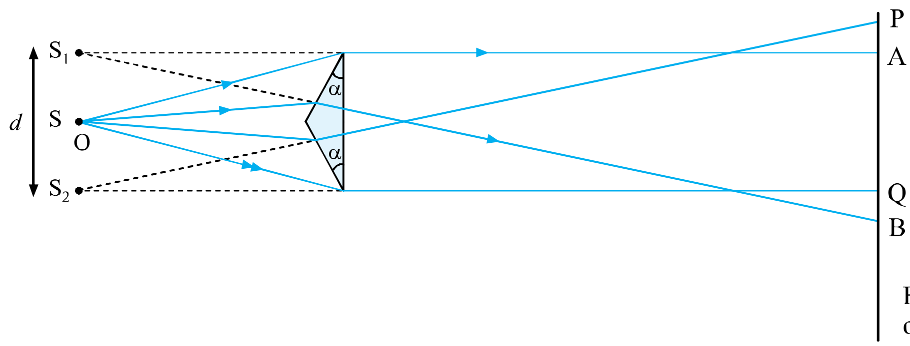
(iii) Graph of Intensity versus distance x from O in double slit experiment :

$$I = I_{max}\cos^2\frac{\pi x}{\omega}$$



FRESNEL'S BIPRISM Section - 3

Fresnel's biprism experiment is a method to produce an interference pattern by forming two virtual images of a source S with the help of a birprism. The biprism is a glass prism of very small base angles $\left(\alpha = \frac{1^\circ}{3}\right)$



Fringe pattern formed on region AQ.

The light from source S passes through the prism and gets refracted. The refracted rays emerging from upper half create a virtual image at S_1 while the rays emerging from lower half of biprism create another virtual image at S_2. The light reaching the screen can now be imagined as coming from S_1 (to the portion AB) and S_2 (to the portion PQ)

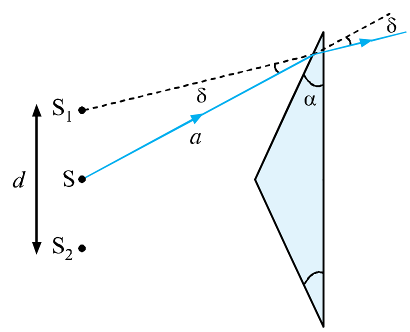
Comparing the situation geometrically with the double-slit experiment,

fringe width $= \omega = \frac{D\lambda}{d}$ where $d = S_1 S_2$ and

D = distance of screen from line joining $S_1 S_2$

λ = light wavelength.

Calculation of *d* :



Let a = distance of source S from biprism.

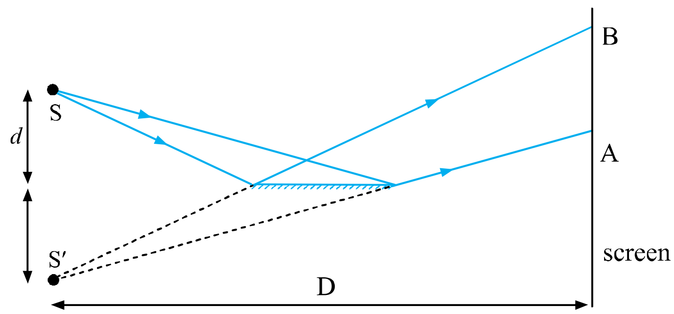
δ = deviation $= (\mu - 1)\,\alpha$ (for small-angled prisms)

$d = 2\,a\,\delta$

$\Rightarrow \quad d = 2a\,(\mu - 1)\,\alpha$

LLOYD'S MIRROR EXPERIMENT Section - 4

In Lloyd's mirror method, the coherent sources required to produce an interference pattern are formed by a real light source S and its virtual image S′ in a plane mirror.



The screen receives light directly from source S and also after reflection from mirror. The reflected light can be imagined as coming from the virtual source S′ .

The portion AB on the screen receives light from both S and S′. The fringe pattern is thus created on this portion only.

Here also, fringe width is $\omega = \dfrac{D\lambda}{d}$

where D = distance of S and S′ from screen.

d = separation between S and S′.

Important Note :

As the reflection is taking place on a denser medium's surface, the reflected light undergoes a phase change of π radians. Hence the light from S' (i.e. reflected light) is in opposite phase with light from S.

Hence the conditions [for sources in opposite phase or exactly out of phase] for constructive and destructive interference are :

P = $n\lambda$ for a dark fringe

P = $(2n - 1)\lambda/2$ for a bright fringe.

Consequently, the central fringe is a dark fringe (because path diff is zero.). It can be observed by shifting the screen close to mirror's right edge

Illustration - 3 *In a Young's double slit experiment the angular width of a fringe formed on a distant screen is* 0.1°. *The wavelength of the light used is* 6000Å. *What is the spacing between the slits ?*

SOLUTION :

Angular width $= \dfrac{\omega}{D} = \dfrac{\lambda}{d}$

$\Rightarrow \quad \dfrac{\lambda}{d} = \dfrac{0.1\pi}{180}$

$\Rightarrow \quad d = \dfrac{180 \times 6000 \times 10^{-10}}{0.1\pi} = 3.44 \times 10^{-4} m$

Illustration - 4 *A two slit Young's interference experiment is done with monochromatic light of wavelength 6000Å. The slits are 2mm apart and the fringes are observed on a screen placed 10 cm away from the slits and it is found that the interference pattern shifts by 5mm when a transparent plate of thickness 0.5 mm is introduced in the path of one of the slits. What is the refractive index of the plate ?*

SOLUTION :

$\lambda = 6000\text{Å}$, $d = 0.2\ cm$, $D = 10\ cm$

Plate thickness $= t = 0.05\ cm$. Shift $= 0.5\ cm$

$$\text{Shift} = \frac{D}{d}(\mu - 1)\,t$$

$$\Rightarrow \quad 0.5 = \frac{10}{0.2}(\mu - 1)0.05 \quad \Rightarrow \quad \mu = 1.2$$

Illustration - 5 *In a Fresnel's biprism experiment, a narrow slit illuminated by monochromatic light of wavelength 6000Å is placed 5cm behind the birpism. A screen is placed at a distance of 1m from the biprism on the other side. A convex lens placed 25 cm from the biprism and 75 cm from the screen forms the real images of coherent sources on the screen. These images are found 0.3 cm apart from each other. The lens is then removed.*

(i) *What is the fringe width of the interference pattern on screen ?*

(ii) *If exactly 20 fringes are observed on the screen, what is be length occupied by the fringe pattern on the screen ?*

SOLUTION :

$D = a + \ell = 105\ cm$

$\lambda = 6 \times 10^{-5}\ cm$

For the convex lens,

$S_1\ S_2$ are the objects

$\Rightarrow \quad u = a + 25 = 30\ cm$

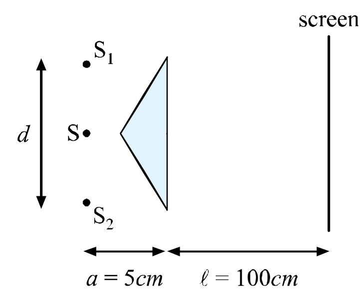
and $\quad v = 75\ cm$

size of object $= S_1\ S_2 = d$

size of image $= 0.3\ cm$

$$\Rightarrow \quad \text{magnification} = \frac{0.3}{d} = \frac{75}{30}$$

$$\Rightarrow \quad d = \frac{3}{25}\ cm = 1.2\ mm.$$



$$\text{Fringe width} = \omega = \frac{D\lambda}{d} = \frac{105 \times 6 \times 10^{-5}}{0.12}$$

$$= 5.25 \times 10^{-2}\ cm$$

Length of fringe pattern $= \omega \times$ no of fringes

$$= 20\omega = 1.05\ cm.$$

Illustration - 6 *A beam of light constituting of two wavelengths 6500 and 5200Å is used to obtain interference fringes in a Young's double slit experiment :*

(i) *Find the distance of the third bright fringe on the screen from the central maximum for the wavelength 6500Å.*

(ii) *What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide ?*

The distance between the slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm.

SOLUTION : $D = 120\ cm$, $d = 0.2\ cm$

Let $\lambda_1 = 6500\ \text{Å}$ and $\lambda_2 = 5200\text{Å}$.

(i) Distance of nth bright fringe from central maximum

$$= \frac{nD\lambda}{d}$$

$\Rightarrow$ Distance of third bright fringe

$$= \frac{3D\lambda_1}{d} = \frac{3 \times 120 \times 6500 \times 10^{-8}}{0.2} = 0.117\, cm.$$

(ii) $\omega_1 = \frac{D\lambda_1}{d}$ and $\omega_2 = \frac{D\lambda_2}{d}$

Let the mth bright fringe due to λ_1 and the nth bright fringe due to λ_2 coincide at a distance x from central maximum.

$\Rightarrow$ $x = m\frac{D\lambda_1}{d}$ and $x = n\frac{D\lambda_2}{d}$

$\Rightarrow$ $m\lambda_1 = n\lambda_2 \Rightarrow 6500\, m = 5200\, n$

$\Rightarrow$ $\frac{m}{n} = \frac{4}{5}$

$\Rightarrow$ 4th bright fringe due to λ_1 coincides with 5th bright fringe due to λ_2 at distance

$$x = \frac{4D\lambda_1}{d} = \frac{4 \times 120 \times 6500 \times 10^{-8}}{0.2}$$

$= 0.156\, cm.$

Alternative Method :

$\frac{\omega_1}{\omega_2} = \frac{\lambda_1}{\lambda_2} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{6500}{5200}$

$\Rightarrow$ $\frac{\omega_1}{\omega_2} = \frac{5}{4} \Rightarrow 4\omega_1 = 5\omega_2$

$\Rightarrow$ Distance of 4th bright fringe due to λ_1

= distance of 5th bright fringe due to λ_2

$\Rightarrow$ $x = 4\omega_1 = 5\omega_2 \Rightarrow x = \frac{4D\lambda_1}{d} = 0.156$ cm.

Illustration - 7 *The path difference at a certain point on screen in a double-slit experiment is one-eighth of the wavelength. Find the ratio of intensity at this point and the intensity at central maximum.*

SOLUTION :

At central maximum, constructive interference occurs.

$\Rightarrow$ $I_{centre} = I_{max}$ $(\Delta\phi = 0)$

At the given point,

$$\Delta\phi = \frac{2\pi}{\lambda}(path\ diff.) = \frac{2\pi}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}$$

$$I = I_{max}\cos^2\frac{\Delta\phi}{2}$$

$\Rightarrow$ $I = I_{max}\cos^2\left(\frac{\pi}{8}\right)$

$\Rightarrow$ $\frac{I}{I_{max}} = \frac{1 + 1/\sqrt{2}}{2} = \frac{\sqrt{2}+1}{2\sqrt{2}}$

Illustration - 8 *A double-slit arrangement produces fringes for λ = 5890 Å that are 0.4° apart. What is the angular width if the entire arrangement is immersed in water ? (μ_w = 4/3)*

SOLUTION :

Angular width $= \omega_0 = \lambda/d$

$\Rightarrow$ $\frac{0.4\pi}{180} = \frac{\lambda_{air}}{d}$ and $\frac{\theta\pi}{180} = \frac{\lambda}{d}$

Let θ = angular width in water

$\Rightarrow$ $\frac{\theta}{0.4} = \frac{\lambda}{\lambda_a}$

By definition of refraction index,

$${}_a\mu_\omega = \frac{c_a}{c_\omega} = \frac{\lambda_a}{\lambda_\omega}$$

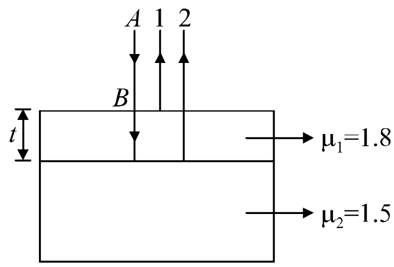
$\Rightarrow$ $\frac{\lambda_a}{\lambda_\omega} = \frac{4}{3} \Rightarrow \frac{\theta}{0.4} = \frac{3}{4}$

$\Rightarrow$ $\theta = \frac{3}{10} = 0.3$ degrees

Illustration - 9 *A glass plate of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If $\lambda = 648$ nm, obtain the least value of t for which the ray interfere constructively.*

SOLUTION :

Incident ray AB is partly reflected as ray 1 from the upper surface and partly reflected as ray 2 from the lower surface of the layer of thickness t and refractive index $\mu_1 = 1.8$ as shown in figure. Path difference between the two ray would by



$$\Delta x = 2\mu_1 t = 2(1.8)\, t = 3.6\, t$$

Ray 1 is reflected from a denser medium, therefore, it undergoes a phase change of π, whereas the ray 2 gets reflected from a rarer medium, therefore, there is no change in phase of ray 2

Hence, phase difference between ray 1 and 2 due to reflection would be $\Delta\phi = \pi$. Therefore, condition of constructive interference will be

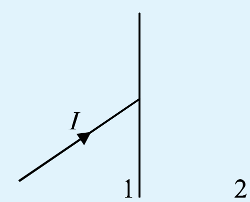
$$\Delta x = \left(n - \frac{1}{2}\right)\lambda \text{ where } n = 1, 2, 3 \ldots.$$

or $$3.6t = \left(n - \frac{1}{2}\right)\lambda$$

Least value of t is corresponding to $n = 1$ or

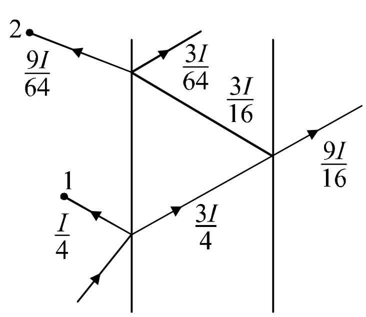
$$t_{\min} = \frac{\lambda}{2 \times 3.6} \quad \text{or} \quad t_{\min} = \frac{648}{7.2} nm = 90 nm$$

Illustration - 10 *A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown in figure. Another identical glass plate is kept close to the first one and parallel to it. Each glass plate reflects 25 per cent of the light incident on it and trasmits the remaining. Find the ratio of the minimum and maximum intensities in the interference pattern formed by the two beams obtained after one reflection at each plate.*



SOLUTION :

Each plate reflects 25% and tansmits 75%.



Incident beam has an intensity I. This beam undergoes multiple reflections and refractions. The corresponding intensity after each reflection and refraction (transmission) are shown in figure. Interference pattern is to take place between rays 1 and 2.

$$I_1 = \frac{I}{4} \text{ and } I_2 = 9I/64$$

$$\therefore \quad \frac{I_{\min}}{I_{\max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2 = \frac{1}{49}$$

IN-CHAPTER EXERCISE

1. *The distance from a Fresnel biprism to a narrow slit and a screen are equal to* 25 *cm and* 75 *cm respectively. The refracting angle of the glass biprism is* $0^\circ\ 20'$. *Find the wavelength of light if the width of the fringe on screen is* 0.55 *mm. Refractive index of glass is* 1.5.

2. *In double slit arrangement, the slits are illuminated by a light of wavelength* 600 *nm. Find the distance of the first point on the screen from the central maximum, where the intensity is* 75% *of the central maximum. The distance between the slits is* 0.25 *cm and the distance of slits from the screen is* 120 *cm.*

3. *Find the ratio of the intensity at the centre of a bright fringe to the intensity at a point one-quarter of the distance between the two fringes from the centre. The amplitudes of interfering sources may be taken equal.*

4. *In an interference pattern the ratio between maximum and minimum intensities is* 36 : 1. *Find the ratio between the amplitudes and intensities of the two interfering waves.*

5. *A thin sheet of glass* ($\mu = 1.5$) *of* 6 *micron thickness introduced in the path of one of the interfering beams in double slit arrangement shifts the central fringe to a position normally occupied by the fifth fringe. Find the wavelength of light used.*

6. *A glass plate* 12×10^{-3} *mm thick is placed in the path of one of the interfering beams in double slit arrangement using monochromatic light of wavelength* 6000Å. *If the central band shifts a distance equal to width of* 10 *bands. Find the* μ *of glass. What is the thickness of the plate of diamond of* $\mu = 2.5$ *that has to be introduced in the path of second beam to bring the central band to original position ?*

7. *In a double slit arrangement, fringes are produced using a light of wavelength* 4800Å. *One slit is covered by thin glass plate of* $\mu = 1.4$ *and other slit by another glass plate of same thickness and of* $\mu = 1.7$. *On doing so the central bright fringe shifts to a position originally occupied by the fifth fringe from the centre. Find the thickness of the glass plate.*

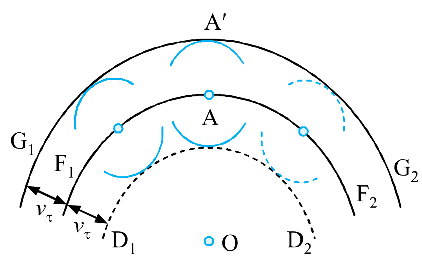
HUYGENS PRINCIPLE Section - 5

If we have a point source emitting waves uniformly in all directions, then the locus of points which have the same amplitude and vibrate in the same phase are spheres and we have what is known as a spherical wave.

Locus of points, which oscillate in phase is called a wavefront ; thus a wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outwards from the source is called the speed of the wave. The energy of the wave travels in a direction perpendicular to the wavefront.

Now, if we know the shape of the wavefront at $t = 0$, then Huygens principle allows us to determine the shape of the wavefront at a later time τ.

Each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave. These wavelets emanating from the wavefront are usually referred to as secondary wavelets and if we draw a common tangent to all these spheres we obtain the new position of the wavefront at a later time.

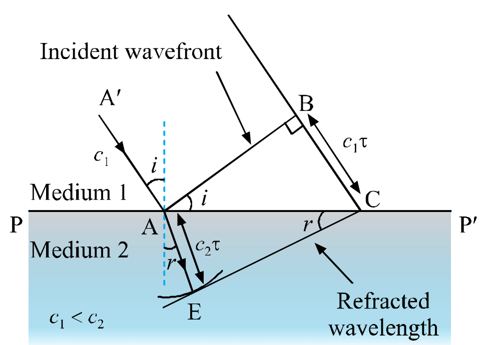


F_1F_2 represents the spherical wavefront (with O as centre) at $t = 0$. The envelope of the secondary wavelets emanating from F_1F_2 produces the forward moving wavefront G_1G_2. The backwave D_1D_2 does not exist.

Refraction of a plane wave

Let c_1 and c_2 represent the speed of light in medium 1 and medium 2, respectively. We assume a plane wavefront AB propagatting in the direction A′A incident on the interface at an angle i as shown in the figure. Let τ be the time taken by the wavefront to travel the distance BC. Thus,

$$BC = c_1\tau$$



A plane wave AB is incident at an angle i on the surface PP′separating medium 1 and medium 2. The plane wave undergoes refraction and CE represents the refracted wavefront. The figure corresponds to $c_2 < c_1$ so that the refracted waves bends towards the normal.

In order to determine the shape of the refracted wavefront, we draw a sphere of radius $c_2\tau$ from the point A in the second medium (the speed of the wave in the second medium is c_2). Let CE represent a tangent plane drawn from the point C on to the sphere. Then, AE = $c_2\tau$ and CE would represent the refracted wavefront. If we now consider the triangles ABC and AEC, we readily obtain

$$\sin i = \frac{BC}{AC} = \frac{c_1\tau}{AC}$$

and $$\sin r = \frac{AE}{AC} = \frac{c_2\tau}{AC}$$

where i and r are the angles of incidence and refraction, respectively.

Thus we obtain

$$\frac{\sin i}{\sin r} = \frac{c_1}{c_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{BC}{AE} = \frac{c_1}{c_2}$$

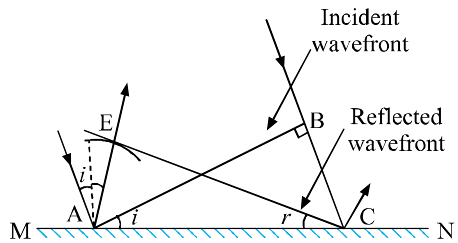
Reflection of a plane wave by a plane surface

We next consider a plane wave AB incident at an angle i on a reflecting surface MN. If c represents the speed of the wave in the medium and if τ represents the time taken by the wavefront to advance from the point B to C then the distance

$$BC = c\tau$$

In order the construct the reflected wavefront we draw a sphere of radius $c\tau$ from the point A as shown in figure. Let CE represent the tangent plane drawn from the point C to this sphere. Obviously

$$AE = BC = c\tau$$

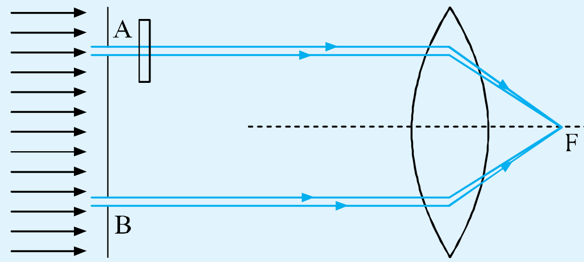


Reflection of a plane wave AB by the reflection surface MN.
AB and CE represent incident and reflected wavefronts.

If we now consider the triangles EAC and BAC we will find that they are congruent and therefore, the angles i and r (as shown in figure) would be equal. This is the law of reflection.

SUBJECTIVE SOLVED EXAMPLES

Example - 1 *In a modified Young's double slit experiment, a mono-chromatic uniform and parallel beam of light of wavelength 6000Å and intensity $10/\pi\ Wm^{-2}$ is incident normally on two circular apertures A and B of radii 0.001 m and 0.002m respectively. A perfect transparent film of thickness 2000Å and refractive index 1.5 for the wavelength 6000Å is placed in front of aperture A. Calculate the power received at the focal spot F of the lens. The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot.*



SOLUTION :

P_1 = Power transmitted through

$$A = \frac{10}{100}\left[\frac{10}{\pi}\right]Wm^{-2}\,\pi\,(0.001\,m)^2$$

P_2 power transmitted through

$$B = \frac{10}{100}\left[\frac{10}{\pi}\right]Wm^{-2}\times\pi\,(0.002)^2\,m^2 = 4\times10^{-6}\text{W}$$

$\Delta\phi$ = phase difference introduced by film.

$$= \frac{2\pi}{\lambda}\text{(path difference introduced)}$$

$$= \frac{2\pi}{\lambda}(\mu-1)\,t = \frac{\pi}{3}\text{ radians}$$

P = power received at F

$$\Rightarrow \quad P = P_1 + P_2 + 2\sqrt{P_1P_2}\cos\Delta\phi$$

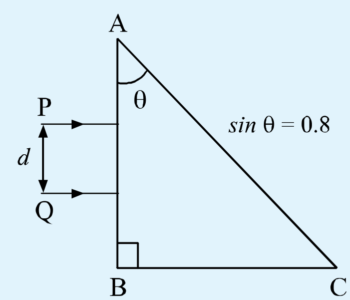
$$= 10^{-6} + 4\times10^{-6} + \sqrt{4\times10^{-12}}\cos\frac{\pi}{3}$$

$$= 7\times10^{-6}W$$

Example - 2 *Two parallel beams of light P and Q (separation d) containing radiations of wavelengths 4000 Å and 5000Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown. The refractive index of the prism as a function of wavelength is given by the relation,*

$$\mu(\lambda) = 1.2 + (b/\lambda^2)$$

where λ is in Å and b is a positive constant. The value of b is such that the condition for total internal reflection at face AC is just satisfied for one wavelength and is not satisfied for the other.



(a) *Find the value of b*

(b) *Find the deviation of the beams transmitted through AC.*

(c) *A convergent lens is used to bring these transmitted beams into focus. If the intensities of the upper and lower beams, immediately after transmission from the face AC, are 4I and I respectively, find the resultant intensity at the focus.*

SOLUTION :

For smaller wavelength λ, μ is more and hence there are more chances of total internal reflection.

$\Rightarrow \quad \lambda = 4000\text{Å}$ is just reflected at AC

$$\Rightarrow \quad \mu_{4000} = \frac{1}{\sin\theta} = \frac{1}{0.8}$$

$$\Rightarrow \quad 1.2 + \frac{b}{(4000)^2} = \frac{1}{0.8} \quad \Rightarrow \quad b = 8\times10^5\ \text{Å}^2.$$

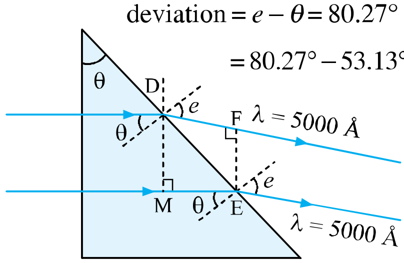
(b) $\mu_{5000} = 1.2 + \frac{8 \times 10^5}{(5000)^2} = 1.232$

At face AC, $\frac{\sin\theta}{\sin e} = \frac{1}{1.232}$

$\Rightarrow \quad \sin e = 0.9856 \quad \Rightarrow \quad e = 80.27°$

deviation $= e - \theta = 80.27° \sin^{-1} 0.8$

$= 80.27° - 53.13° = 27.14°.$



(c) Optical Path difference between 1 and 2 is :

$P = x_2 - x_1$

$= (ME)\,\mu - DF$

$= \mu d \tan\theta - \frac{d}{\cos\theta} \sin e$

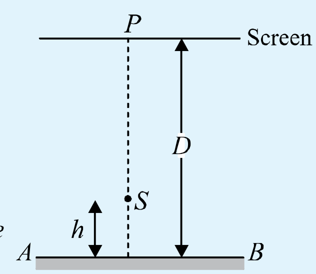
$= \mu d \tan\theta - d \tan\theta \frac{\sin e}{\sin\theta}$

$= \mu d \tan\theta - \mu d \tan\theta = 0$

Hence the resultant intensity is :

$I = I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 9I$

Example - 3 *A point source S emitting light of wavelength 600 nm is placed at a very small height h above a flat reflecting surface AB (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance D from it.*



(a) *What is the shape of the interference fringes on the screen?*

(b) *Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P.*

(c) *If the intensity at point P corresponds to a maximum, calculate the minimum distance through which the source S should be shifted so that the intensity at P again becomes maximum.*

SOLUTION :

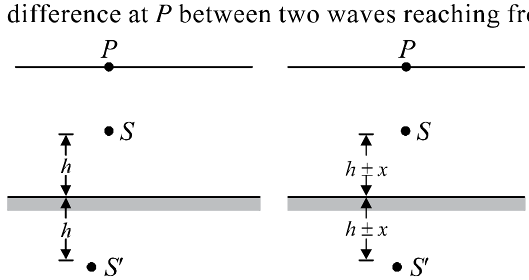
(a) Shape of the interference fringes will be circular.

(b) Intensity of light reaching on the screen directly from the source $I_1 = I_0$ (say) and intensity of light reaching on the screen after reflecting from the mirror is $I_2 = 36\%$ of $I_0 = 0.36 I_0$.

$\therefore \quad \frac{I_1}{I_2} = \frac{I_0}{0.36 I_0} = \frac{1}{0.36}$ or $\sqrt{\frac{I_1}{I_2}} = \frac{1}{0.6}$

$$\therefore \quad \frac{I_{\min}}{I_{\max}} = \frac{(\sqrt{I_1} - \sqrt{I_2})^2}{(\sqrt{I_1} + \sqrt{I_2})^2} = \frac{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2} = \frac{\left(\frac{1}{0.6} - 1\right)^2}{\left(\frac{1}{0.6} + 1\right)^2} = \frac{1}{16}$$

(c) Intially path difference at P between two waves reaching from S and S' is $2h$.



Initial Final

As there is a phase differecne of π due to reflection from mirror. Therefore, for maximum intensity at P :

$$2h = \left(n - \frac{1}{2}\right)\lambda \quad \text{. . . (i)}$$

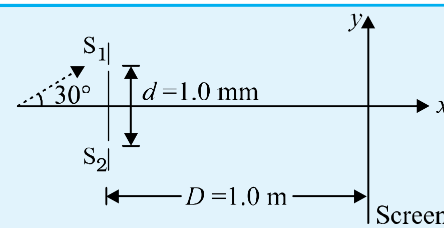
Now, let the source S is shifted by x. The path difference will be $2h + 2x$ or $2h - 2x$. For maximum intensity at P again:

$$2h + 2x = \left[n + 1 - \frac{1}{2}\right]\lambda \quad \text{. . . .(ii)} \qquad \text{or} \qquad 2h - 2x = \left[n - 1 - \frac{1}{2}\right]\lambda \quad \text{. . . . (iii)}$$

Solving Eqs. (i) and (ii) or Eqs. (i) and (iii), we get :

$$x = \frac{\lambda}{2} = \frac{600}{2} = 300\ nm$$

Example - 4 *A coherent parallel beam of microwaves of wavelength $\lambda = 0.5$ mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in the figure.*



(a) *If the incident beam falls normally on the double slit apparatus, find the y-coordinates of all the interfrence minima on the screen.*

(b) *If the incident beam makes an angle of 30° with the x-axis (as in the dotted arrow shown in figure), find the y-coordinates of the first minima on either side of the central maximum.*

SOLUTION :

Given, $\lambda = 0.5$ mm, $d = 1.0$ mm, $D = 1$ m

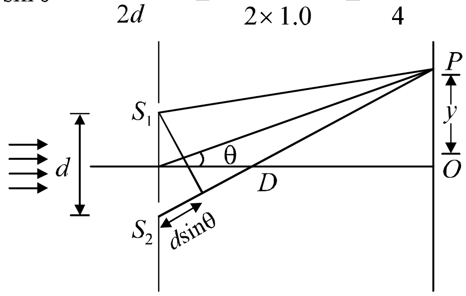
(a) When the incident beam falls normally :

Path difference between the two rays S_2P and S_1P is

$\Delta x = S_2 - S_1P = d \sin\theta$

For minimum intensity, $d \sin\theta = (2n-1)\frac{\lambda}{2}$, where n = 1, 2, 3 .....

$$\text{or} \quad \sin\theta = \frac{(2n-1)\lambda}{2d} = \frac{(2n-1)0.5}{2\times 1.0} = \frac{2n-1}{4}$$



As $\sin\theta \le 1$ therefore $\frac{(2n-1)}{4} \le 1$ or $n \le 2.5$

So, n can be either 1 or 2.

When $n = 1$, $\sin\theta_1 = \frac{1}{4}$ or $\tan\theta_1 = \frac{1}{\sqrt{15}}$

When $n = 2$, $\sin\theta_2 = \frac{3}{4}$ or $\tan\theta_2 = \frac{3}{\sqrt{7}}$

$\therefore \quad y = D\tan\theta = \tan\theta \quad (D = 1\ m)$

So, the position of minima will be

$$y_1 = \tan\theta_1 = \frac{1}{\sqrt{15}}\ m = 0.26\ m$$

$$y_2 = \tan\theta_2 = \frac{3}{\sqrt{7}}\ m = 1.13\ m$$

And as minima can be on either side of centre O.

Therefore there will be four minimas at positions ± 0.26 m and ± 1.13 m on the screen.

(b) When $\alpha = 30°$, path difference between the rays before reaching S_1 and S_2 is

$$\Delta x_1 = d \sin\alpha = (1.0)\sin 30° = 0.5\ mm = \lambda$$

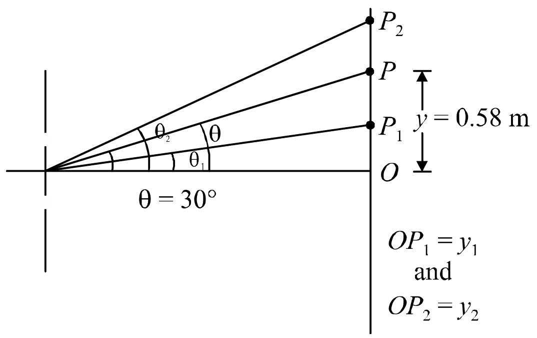
So, there is already a path difference of λ between the rays.

Position of central maximum Central maximum is defined as a point where net path difference is zero.

So, $\Delta x_1 = \Delta x_2$

or $d\sin\alpha = d\sin\theta \Rightarrow \theta = \alpha = 30°$

or $\tan\theta = \frac{1}{\sqrt{3}} = \frac{y_0}{D} \Rightarrow y_0 = 0.58\,m \quad (D = 1m)$



At point P, $\Delta x_1 = \Delta x_2$, Above point P $\Delta x_2 > \Delta x_1$ and Below point P $\Delta x_1 > \Delta x_2$

Now, let P_1 and P_2 be the minimas on either side of cental maxima. Then, for P_2

$$\Delta x_2 - \Delta x_1 = \frac{\lambda}{2}$$

$$\Rightarrow \Delta x_2 = \Delta x_1 + \frac{\lambda}{2} = \lambda + \frac{\lambda}{2} = \frac{3\lambda}{2}$$

$$\Rightarrow d\sin\theta_2 = \frac{3\lambda}{2}$$

or $\sin\theta_2 = \frac{3\lambda}{2d} \frac{(3)(0.5)}{(2)(1.0)} = \frac{3}{4}$

$\therefore \tan\theta_2 = \frac{3}{\sqrt{7}} = \frac{y_2}{D} \Rightarrow y_2 = \frac{3}{\sqrt{7}} = 1.13\,m$

Similarly for P_1

$$\Delta x_1 - \Delta x_2 = \frac{\lambda}{2}$$

$$\Rightarrow \Delta x_2 = \Delta x_1 - \frac{\lambda}{2} = \lambda - \frac{\lambda}{2} = \frac{\lambda}{2}$$

$$\Rightarrow d\sin\theta_1 = \frac{\lambda}{2}$$

or $\sin\theta_1 = \frac{\lambda}{2d} = \frac{(0.5)}{(2)(1.0)} = \frac{1}{4}$

$\therefore \tan\theta_1 = \frac{1}{\sqrt{15}} = \frac{y_1}{D}$

$\Rightarrow y_1 = \frac{1}{\sqrt{15}} m = 0.26\,m$

Therefore, y-coordinates of the first minima on either side of the central maximum are y_1 = 0.26 and $y_2 = 1.13\,m$.

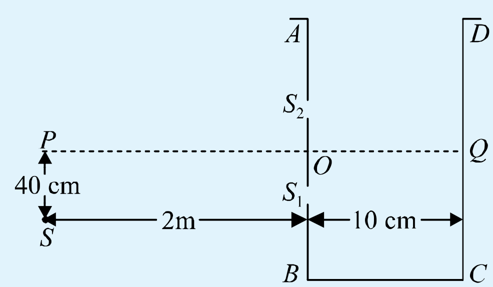
Note : In this problem $\sin\theta \approx \tan\theta \approx \theta$ is not valid as θ is large.

Example - 5 *A vessel ABCD of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passsing through O, the middle point of S_1 and S_2.*

A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure alongside.

(a) *Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ. Now, a liquid is poured into the vessel and filled upto OQ.*

(b) *If the central bright fringe is found to be at Q, calculate the refractive index of the liquid.*



SOLUTION :

(a) Given, $y_1 = 40\,cm$, $D_1 = 2m = 200\,cm$, $D_2 = 10\,cm$

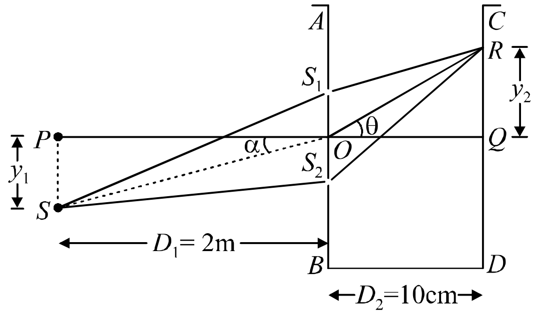
ΔX at R will be zero if $\Delta X_1 = \Delta X_2$

or $d \sin\alpha = d \sin\theta$

or $\alpha = \theta$ or $\tan\alpha = \tan\theta$

$\Rightarrow \dfrac{y_1}{D_1} = \dfrac{y_2}{D_2}$

or $y_2 = \dfrac{D_2}{D_1} \cdot y_1 = \left(\dfrac{10}{200}\right)(40)\ cm = 2cm$



(b) The central bright fringe will be observed at point Q, if the path difference created by the liquid slab of thickness $t = 10cm$ or $100\ mm$ is equal to ΔX_1, so that the net path differecne at Q becomes zero.

So, $(\mu - 1)\,t = \Delta X_1$ or $(\mu - 1)(100) = 0.1$ or $\mu = 1.0016$

MISCELLANEOUS EXERCISE

Choose the correct options for each of the following questions. Questions marked with * may have more than one correct options.

1. If the wavelength of light used in the young's double slit experiment is λ and if d is the width of the slit, what is the angular width of the central maximum ?

(A) $sin^{-1}(\lambda/d)$ **(B)** $2sin^{-1}(\lambda/d)$

(C) $sin^{-1}(2\lambda/d)$ **(D)** $sin^{-1}(\lambda/2d)$

2. White light is used to illuminate the two slits in Young's double slit experiment. The separation between the slits is d and the distance between the screen and the slit is D (>> d). At a point on the screen directly in front of one of the slits, certain wavelength are missing. The missing wavelengths are (here m = 0, 1, 2, . . . is an integer)

(A) $\lambda = \dfrac{d^2}{(2m+1)\,D}$ **(B)** $\lambda = \dfrac{(2m+1)\,d^2}{D}$

(C) $\lambda = \dfrac{d^2}{(m+1)\,D}$ **(D)** $\lambda = \dfrac{(m+1)\,d^2}{D}$

3. In an interference experiment, 20th order maximum is observed at a point on the screen when light of wavelength 480 nm is used. If this light is replaced by light of wavelength 600 nm, the order of the maximum at the same point will be :

(A) 16 **(B)** 14 **(C)** 12 **(D)** 10

4. When a ray of light goes from a denser into a rarer medium

(A) The wavelength of light is decreased

(B) The frequency of light is increased

(C) The speed of light is increased

(D) The light undergoes a phase change of π

5. In a Young's double-slit experiment, the central bright fringe can be identified

(A) as it has greater intensity than the other bright fringes

(B) as it is wider than the other bright fringes

(C) as it is narrower than the bright fringes

(D) by using white light instead of monochromatic light

6. In a Young's double-slit experiment, let β be the fringe width, and let I_0 be the intensity at the central bright fringe. At a distance x from the central bright fringe, the intensity will be :

(A) $I_0 \cos\left(\dfrac{x}{\beta}\right)$ **(B)** $I_0 \cos^2\left(\dfrac{x}{\beta}\right)$

(C) $I_0 \cos^2\left(\dfrac{\pi x}{\beta}\right)$ **(D)** $\left(\dfrac{I_0}{4}\right)\cos^2\left(\dfrac{\pi x}{\beta}\right)$

ANSWERS TO IN-CHAPTER EXERCISES

1. 8×10^{-7} m **2.** 0.288 mm **3.** 2 : 1 **4.** 7 : 5, 49 : 25 **5.** 6000 Å

6. $\mu = 1.5$; $t = 4 \times 10^{-3}$ mm **7.** 8×10^{-3} mm

ANSWERS TO MISCELLANEOUS EXERCISE

1. A **2.** A **3.** A **4.** C **5.** D **6.** C